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# SPECTRUM OF NORMAL OSCILLATIONS OF A DISLOCATION ENSEMBLE IN A VISCOPLASTIC MEDIUM

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*Dynamic equations of a dislocation ensemble are derived within the framework of the gauge model of an elastic body with defects. The governing relation between the defect field and the material continuum is obtained with allowance for the analogy between the equations obtained and Maxwell's electrodynamic equations. In the phenomenological theories of plasticity, this relation corresponds to the definition of a viscoplastic body. The dispersion relations and configurations of the normal oscillations of a dislocation ensemble in a viscoplastic medium are calculated.*

**Introduction.** The continual theory of defects studies imperfections of a crystal lattice within the framework of continuum mechanics. A crystal with dislocations is usually modeled by an elastic body with internal stresses that do not depend on the applied surface and body forces. An advantage of this model is that methods of the theory of elasticity can be used to calculate the displacement and stress fields produced by both isolated defects and defects characterized by a continuous dislocation-density tensor [1–3]. In some papers [3, 4], along with the static characteristic of a dislocation ensemble (dislocation-density tensor), a dynamic quantity — the dislocation flux density tensor — is considered.

The complete system of equations of the continual theory of dislocations includes the geometrical equations of an elastic continuum with defects, Hooke's law, and the equations of motion of the medium [4]. This system of equations enables one to study the dynamics of stresses and velocities of an elastic medium with specified dislocation density and dislocation flux. The further development of the continual theory of defects is based on the ideas and methods of the gauge field theories [5–7]. The closed system of dynamic equations for an elastic continuum with internal stresses [6] and the system of dynamic equations for an ensemble of defects [7] can be constructed within the framework of this approach. The former equations were used to study the normal-oscillation spectra of an elastic medium with defects and to construct a phenomenological generalization of the model with allowance for energy dissipation [8].

In this paper, the dynamic equations of a dislocation ensemble are considered and the dispersion relations of a defect field in a viscoplastic medium are calculated.

**Description of the Model.** In accordance with [6, 7], an elastic body with defects can be studied on the basis of a model of a mixture of two continua, one of which is the material medium and the other is the defect field. The Lagrangian density of this model is written as

$$L = \rho \boldsymbol{V} \cdot \boldsymbol{V} - \beta : C : \beta + BI : I - S\alpha : \alpha, \qquad (1)$$

where $\boldsymbol{V} = \partial \boldsymbol{u}^{\rm ext}/\partial t + \boldsymbol{V}^{\rm int}$ is the effective velocity of the medium, $\beta = \nabla \boldsymbol{u}^{\rm ext} + \beta^{\rm int}$ is the effective elastic distortion, which depends on the elastic displacements $\boldsymbol{u}^{\rm ext}$ caused by external actions, the elastic distortion due to the material defects $\beta^{\rm int}$, and the displacement velocity $\boldsymbol{V}^{\rm int}$ due to the motion of defects, $\alpha = \nabla \times \beta^{\rm int}$

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is the dislocation-density tensor, $I = \partial\beta^{\rm int}/\partial t - \nabla \boldsymbol{V}^{\rm int}$ is the dislocation flux tensor, $\rho$ is the density, $C$ is the elastic-modulus tensor of rank 4, and $B$ and $S$ are constants. Symbols "$\cdot$" and ":" denote scalar convolution with respect to one or two indices and symbol "$\times$" denotes the vector product. The first two terms of the Lagrangian (1) characterize the elastic continuum subjected to the action of external loads and material defects, and the last two terms characterize the defect field.

Varying the Lagrangian density (1) with respect to the independent variables $\boldsymbol{u}^{\rm ext}$, $\beta^{\rm int}$, and $\boldsymbol{V}^{\rm int}$, we obtain the dynamic equations for the elastic medium with defects:

$$\frac{\partial \boldsymbol{P}}{\partial t} = \nabla\sigma, \tag{2}$$

$$B\nabla \cdot I = -\boldsymbol{P}, \qquad S\nabla \times \alpha = -B\frac{\partial I}{\partial t} - \sigma. \tag{3}$$

Here $\boldsymbol{P} = \rho\boldsymbol{V}$ is the effective momentum of the medium and $\sigma = C : \beta$ is the stress tensor. Equations (3) together with the geometrical relations of an elastic continuum

$$\nabla \cdot \alpha = 0, \qquad \frac{\partial\alpha}{\partial t} = \nabla \times I, \tag{4}$$

which express the continuity condition for the dislocation-density tensor and the law of conservation of the Burgers vector, constitute the complete system of dynamic equations of the dislocation ensemble. The equation of dynamic equilibrium (2) is the compatibility condition for the given system. However, Eqs. (2)–(4) are not closed since the relation between the elastic continuum and the defect field is lacking.

Using the formal analogy between the given equations and the Maxwell's electrodynamic equations [9], we can relate the dislocation-flux tensor $I$ with the electric-field strength, the density tensor $\alpha$ with the magnetic-field strength, the effective momentum $\boldsymbol{P}$ with the charge, and the stress tensor $\sigma$ with the current. As a result, we write the relation

$$\sigma = \eta I, \tag{5}$$

which is similar to the relation between an electromagnetic field and material in the case of a homogeneous conducting medium. In the phenomenological theories of plasticity [10], this relation corresponds to the definition of a viscoplastic body, which implies that the coefficient $\eta$ has the meaning of the generalized viscosity of the medium.

**Normal-Oscillation Spectrum of the Dislocation Ensemble in the Viscoplastic Medium.** We consider the propagation of the plane monochromatic wave of the defect field in the viscoplastic medium. We seek a solution of (2)–(5) in the form

$$I = I_0 \exp(iwt - ikx), \qquad \alpha = \alpha_0 \exp(iwt - ikx). \tag{6}$$

Substituting (6) into (2)–(5), we obtain a system of characteristic equations that define 18 branches of the dispersion law:

$$k^2_{1,\dots,14} = \frac{B}{S}w^2, \quad k^2_{15,16} = \frac{B}{S}w^2 - \frac{iw\eta}{S}, \quad k^2_{17,18} = \Big[\frac{B}{S}w^2\Big(\frac{\eta^2}{2B^2} + w^2\Big) - \frac{i\eta w^3}{2S}\Big]\Big/\Big(\frac{\eta^2}{4B^2} + w^2\Big),$$

or, in dimensional form

$$K^2_{1,\dots,14} = \omega^2, \quad K^2_{15,16} = \omega^2 - i\omega, \quad K^2_{17,18} = 2(\omega^2(1 + 2\omega^2) - i\omega^3)/(1 + 4\omega^2). \tag{7}$$

The corresponding dispersion curves for high and low frequencies are shown in Figs. 1 and 2. Since the last two equations of (7) are complex, the corresponding dispersion curves have real and imaginary parts. The real part $K(\omega)$, denoted by $R(\omega)$, defines the reciprocal of the wavelength of the oscillations, and the imaginary part $Z(\omega)$ gives the reciprocal of the depth of penetration of a wave of given frequency, i.e., the absorption

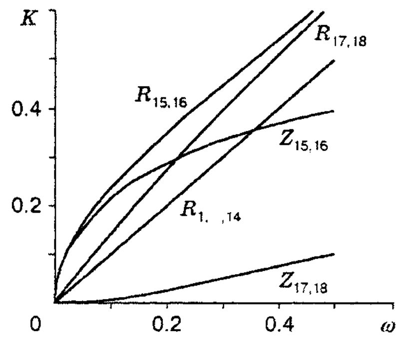


Fig. 1

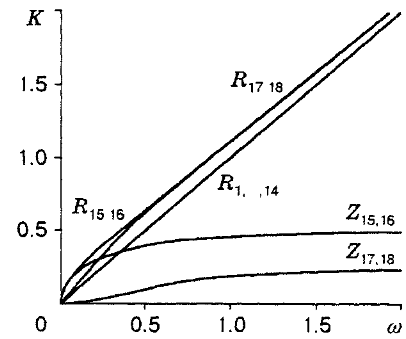


Fig. 2

factor. The curves $R_j(\omega)$ and $Z_j(\omega)$ ($j$ is the branch number) in Figs. 1 and 2 show the dependences of the real and imaginary parts on the frequency.

**Discussion of Results.** The results obtained show that the normal-oscillation spectrum of the dislocation ensemble in the viscoplastic medium consists of three wave groups, one of which ($K_1, \ldots, K_{14}$) propagates at a constant velocity without dissipation, and the other two wave groups ($K_{15}$ and $K_{16}$; $K_{17}$ and $K_{18}$) possess dispersion and dissipation. An analysis of the curves presented in Figs. 1 and 2 shows that the wave dispersion is significant for low frequencies and is almost absent for high frequencies. For high frequencies, the absorption factor is also independent of frequency.

Analysis of the normal-oscillation configurations made it possible to identify each branch of the dispersion law with oscillations of definite quantities. The branches $K_1$, $K_2$, $\ldots$, $K_6$ describe oscillations of the quantities $\alpha_{zy}$, $\alpha_{yz}$, $\alpha_{xx}$, $\alpha_{xy}$, $\alpha_{xz}$, $I_{xx}$. The two pairs $K_7$, $K_8$ and $K_9$, $K_{10}$ describe oscillations of $\alpha_{yx}$, $I_{zx}$, and $I_{xz}$ and $\alpha_{zx}$, $I_{yx}$, and $I_{xy}$, respectively, with the configurations $\alpha_{yx} = I_{zx}\sqrt{B/S}$, $I_{xz} = 0$ and $\alpha_{zx} = I_{yx}\sqrt{B/S}$, $I_{xy} = 0$. The four branches $K_{11}$, $K_{12}$, $K_{13}$, and $K_{14}$ correspond to oscillations of the quantities $I_{yz}$, $I_{zy}$, $\alpha_{yy}$, and $\alpha_{zz}$, for which the following relations hold: $I_{yz} = I_{zy}$, $\alpha_{yy} = I_{zy}\sqrt{B/S}$, and $\alpha_{zz} = I_{zy}\sqrt{B/S}$. The branches $K_{15}$ and $K_{16}$ correspond to oscillations of the quantities $\alpha_{zy}$, $I_{yy}$ and $\alpha_{yz}$, $I_{zz}$, respectively, whose configurations are given by the expression

$$\alpha_{zy} = \sqrt{\frac{B}{2S}\Big(\frac{\sqrt{1+\omega^2}}{\omega} + 1\Big)}\, I_{yy}.$$

A similar relation is valid for the quantities $\alpha_{yz}$ and $I_{zz}$. The branches $K_{17}$ and $K_{18}$ describe oscillations of the quantities $\alpha_{zx}$ and $I_{yx}$ and $\alpha_{yx}$ and $I_{zx}$, for which

$$\alpha_{zx} = \sqrt{\frac{(B/S)\sqrt{(1+2\omega^2)^2+\omega^2} + 1 + 2\omega^2}{1+4\omega^2}}\, I_{yx}.$$

A similar relation is valid for the quantities $\alpha_{yx}$ and $I_{zx}$.

Thus, when the plane defect wave propagates in the viscoplastic medium, oscillations of all components of the dislocation-density tensor can be recorded. Oscillations of the diagonal components of the dislocation flux tensor $I_{yy}$ and $I_{zz}$, which govern motion of defects in a plane that does not coincide with the wave-front plane, attenuate as they propagate. Oscillations of the components $I_{yx}$ and $I_{zx}$, defined by the Burgers-vector flux in the direction of motion of the defect wave in the planes perpendicular to this direction, attenuate as well. There are no oscillations of the flux-tensor components $I_{xy}$ and $I_{xz}$, which correspond to the Burgers-vector flux along the axes perpendicular to the direction of propagation of the wave of defects moving in the wave plane. The above results can be used to analyze results of acoustic emission in media with defects.

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